

WMAPping out Neutrino Masses

Aaron Pierce

*Theoretical Physics Group, Stanford Linear Accelerator Center,
Stanford University, Stanford, California, 94309, USA*

Hitoshi Murayama

*Department of Physics, University of California, Berkeley, California, 94720, USA and
Theory Group, Lawrence Berkeley National Laboratory, Berkeley, California, 94720, USA*

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Recent data from the Wilkinson Microwave Anisotropy Probe (WMAP) place important bounds on the neutrino sector. The precise determination of the baryon number in the universe puts a strong constraint on the number of relativistic species during Big-Bang Nucleosynthesis. WMAP data, when combined with the 2dF Galaxy Redshift Survey (2dFGRS), also directly constrain the absolute mass scale of neutrinos. These results impinge upon a neutrino oscillation interpretation of the result from the Liquid Scintillator Neutrino Detector (LSND). We also note that the Heidelberg–Moscow evidence for neutrinoless double beta decay is only consistent with the WMAP+2dFGRS data for the largest values of the nuclear matrix element.

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I. INTRODUCTION

Evidence for neutrino oscillation has steadily mounted over the last few years, culminating in a picture that presents a compelling argument for finite neutrino masses. The observation of a zenith-angle dependent deficit of ν_μ from cosmic ray showers at Super-Kamiokande [1], provided strong evidence for oscillations in atmospheric neutrinos. Recent results on solar neutrinos at the Sudbury Neutrino Observatory (SNO) [2] and reactor neutrinos at the KamLAND experiment [3], have shed light on the solar neutrino problem. These experiments have provided strong evidence that the solar neutrino problem is solved by oscillations corresponding to the Large Mixing Angle solution [4]. Although clear oscillation data now exist in atmospheric, reactor, and solar neutrino experiments, it remains to determine the significance of the result from the Liquid Scintillator Neutrino Detector (LSND) [5, 6], which claimed evidence for conversion of $\bar{\nu}_\mu$ to $\bar{\nu}_e$ with a Δm_ν^2 of order 1 eV^2 .

While these extraordinary advances in experimental neutrino physics were occurring, a concurrent revolution in experimental cosmology took place. Ushered in by the Boomerang, MAXIMA, and DASI measurements of the acoustic peaks in the Cosmic Microwave Background (CMBR) [7], an era has begun wherein it is possible to make measurements of cosmological parameters with previously unimaginable precision. Most recently, the striking data [8] from the Wilkinson Microwave Anisotropy Probe (WMAP) have vastly improved our knowledge of several fundamental cosmological parameters [9]. Because cosmology would be significantly affected by the presence of light species with masses of order 1 eV , the new WMAP data strongly constrain neutrino masses in this range. We will show this brings cosmology into some conflict with the LSND result in two ways.

First, WMAP determines the baryon to photon ratio

very precisely. This removes an important source of uncertainty in the prediction of Big-Bang Nucleosynthesis (BBN) for the primordial abundance of ^4He . This allows for a strong limit to be placed on the number of relativistic species present at BBN, disfavoring the LSND result. Secondly, WMAP, when combined with data from the 2 degree Field Galactic Redshift Survey (2dFGRS) [10], CBI [11], and ACBAR [12], is able to place stringent limits on the amount that neutrinos contribute to the critical density of the universe. This second constraint results in an upper mass-limit on neutrinos that contradicts the LSND result in all but one “island” of parameter space not ruled out by other experiments. The second constraint also impinges on the recent evidence for neutrinoless double beta decay from the Heidelberg-Moscow experiment [13].

II. THE LSND RESULT

The LSND experiment used decays of stopped anti-muons at the LAMPF facility (Los Alamos) to look for the appearance of anti-electron-neutrinos. They reported the oscillation probability $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) = (0.264 \pm 0.067 \pm 0.045)\%$, representing a 3.3σ signal.

If the result at the LSND experiment were a true indication of oscillations, it would have profound implications for our understanding of neutrinos. Solar and atmospheric neutrinos have already determined two neutrino mass-squared differences to be $\Delta m_{\text{solar}}^2 \sim 10^{-4} \text{ eV}^2$ and $\Delta m_{\text{atm}}^2 \sim 10^{-3} \text{ eV}^2$. However, taking into account the Bugey exclusion region [14], the LSND experiment points to a mass difference (see Figure 1) $\Delta m_{\text{LSND}}^2 > 10^{-1} \text{ eV}^2$. The presence of this completely disparate mass difference necessitates the introduction of a fourth neutrino [44]. Because LEP has determined the number of active neutrino species to be three, this fourth

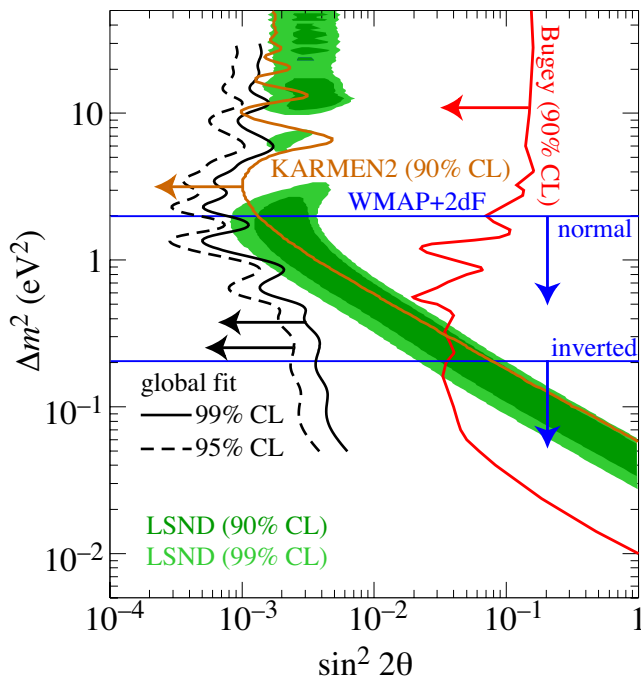


FIG. 1: The LSND Allowed region, with Bugey and Karmen [15] exclusion regions. The constraints from the global fit [16] as well as the limit of [17] from the combination of WMAP and 2dFGRS data are also shown. There are two lines, corresponding to the 3 + 1 (normal) and 1 + 3 (inverted) spectra. The contours from the global fit would, of course, continue on to lower values of Δm^2 , but Ref. [16] did not show this region.

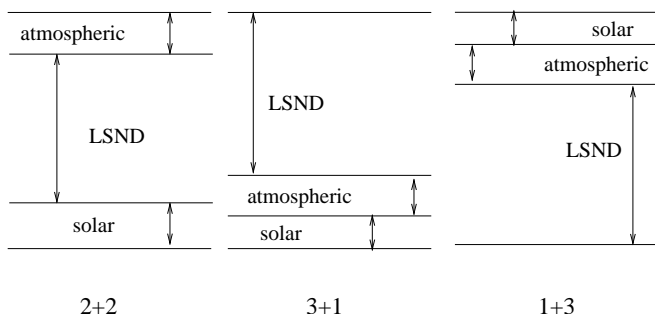


FIG. 2: Sample neutrino spectra in the light of LSND. Different permutations are also possible.

neutrino must be sterile, having extraordinarily feeble couplings to the other particles of the standard model.

The introduction of this fourth neutrino species results in principle in two characteristic types of spectra, 2 + 2 and 3 + 1. Two sample spectra of these types are shown in Figure 2.

However, recent results from SNO [2] and Super-Kamiokande [20], have indicated that the oscillations responsible for the atmospheric and solar neutrino anomalies involve transitions primarily between active neutrinos. This means that it is difficult to put the sterile part of the neutrino in either the solar or atmospheric

pair in the 2 + 2 spectrum. A recent quantitative analysis [19] found this 2 + 2 spectrum to be completely ruled out, while a 3 + 1 spectrum was allowed at the 99% confidence level [16, 19]. The tension for the 3 + 1 spectrum is in large part due to the lack of a signal in short-baseline disappearance experiments such as CDHSW [21] and Bugey. Adding additional sterile neutrinos can only marginally improve this agreement [22]. In the next two sections, we show how this allowed window is further constrained by cosmological considerations.

III. BIG-BANG NUCLEOSYNTHESIS

By measuring the primordial abundance of ^4He , one can place bounds on extra relativistic degrees of freedom at the time of Big Bang Nucleosynthesis (BBN) [23]. These bounds are usually quoted in terms of a number of effective allowed neutrino species, N_ν^{eff} . Additional degrees of freedom tend to increase the expansion rate of the universe, which causes neutrons to freeze out at an earlier time, at a higher abundance. This abundance translates into more primordial ^4He for a given baryon to photon ratio, η . Therefore, knowledge of primordial ^4He abundance along with a separate determination of η places a bound on N_ν^{eff} . On the other hand, for a fixed N_ν^{eff} , a higher η results in a higher abundance for primordial ^4He ; so, incomplete knowledge of η degrades the constraint on N_ν^{eff} .

In the era before precise CMBR measurements, BBN data alone were utilized to set the bound. Measurements of primordial deuterium or lithium were used for the separate determination of η . An aggressive analysis by [24] cited a limit of $N_\nu^{\text{eff}} < 3.4$ at 2σ , and consequently found that LSND data were strongly disfavored by BBN [25]. However, the data for primordial light element abundances were somewhat muddled, with some measurements of lithium and deuterium preferring substantially lower values of η than others. Due to the presence of these data, a conservative bound $N_\nu < 4$ was often taken [26]. In fact, using lithium data alone, [27], found that even $N_\nu^{\text{eff}} = 4.9$ was acceptable at the 95% confidence level.

However, after precise measurements of the CMBR, the situation has changed. The WMAP experiment has determined [9] $\Omega_b h^2 = 0.224 \pm 0.001$, corresponding to an $\eta = 6.5_{-0.3}^{+0.4} \times 10^{-10}$. For the central value above, the expected ^4He abundance, Y_p , is roughly $Y_p = 0.249 + 0.013(N_\nu^{\text{eff}} - 3)$. The status of primordial Helium measurements remains controversial. One helium measurement quotes a value $Y_p = 0.244 \pm 0.002$ [29], while another quotes $Y_p = 0.235 \pm 0.002$ [28]. To deal with the discrepancy in these measurements, the Particle Data Group (PDG) assigns an additional systematic error, taking $Y_p = 0.238 \pm 0.002 \pm 0.005$ [30]. To be completely conservative, we will take the higher helium abundance, and assign to it the additional systematic error of the PDG, namely, we take $Y_p = 0.244 \pm 0.002 \pm 0.005$. Using

the formulae of [31] for ${}^4\text{He}$ in terms of N_ν^{eff} and η , we find $N_\nu^{\text{eff}} < 3.4$ at the 95% (two-sided) confidence level, leaving no room for the extra neutrino of LSND. Using the only slightly less conservative approach of adopting the PDG central value and error, we find $N_\nu^{\text{eff}} < 3.0$ at the 95% (two-sided) confidence level.

Of course, additional systematic errors in the helium abundance measurements may be found. The fact that 3 neutrinos is barely consistent at the 95% confidence level might cause some suspicion that there are unknown systematics at work. However, to get $N_\nu^{\text{eff}} = 4$ at the 95% level would require inflating the errors on the PDG central value dramatically, to $Y_p = 0.238 \pm 0.011$.

It is possible that an asymmetry in the leptons could effectively prevent the oscillation into sterile neutrinos [32]. We find that a large pre-existing asymmetry of $L^{(e)} \sim 10^{-2}$ would be sufficient to suppress the production of sterile neutrinos below the BBN constraint. Here, L^e , represents the total asymmetry felt by electron neutrinos, $L^e = 2L_{\nu_e} + L_{\nu_\mu} + L_{\nu_\tau}$, with $L_{\nu_i} = (N_{\nu_i} - N_{\bar{\nu}_i})/N_\gamma$. Smaller L^e (as low as $\sim 10^{-5}$) can suppress sterile neutrino production, but oscillations tend to erase asymmetries of this size. While a lepton asymmetry of 10^{-2} size does not bias the rate for the processes such as $n + e^+ \rightarrow p + \bar{\nu}_e$ significantly enough to affect BBN, an asymmetry as large as 10^{-1} would. CMBR constraints also cannot exclude the possibility of a lepton asymmetry of 10^{-2} , so this possibility can not be excluded. It does not appear that neutrino oscillations themselves can create this asymmetry [33]. One has to assume that the asymmetry existed before the BBN, possibly generated by a mechanism similar to that in [34].

IV. WEIGHING NEUTRINOS WITH LARGE SCALE STRUCTURE

WMAP has provided an additional constraint on LSND. As noted, for example, in [35], Galactic Surveys provide a powerful tool to constrain the masses of neutrinos. Neutrinos decouple at temperatures well above those at which structure forms. They then free-stream until they become non-relativistic. This tends to smooth out structure on the smallest scales. On scales within the horizon when the neutrinos were still relativistic, the power spectrum of density fluctuations is suppressed as [35]:

$$\frac{\Delta P_m}{P_m} \approx -8 \frac{\Omega_\nu}{\Omega_m} \quad (1)$$

The 2dFGRS experiment used this fact to place a limit on the sum of neutrino masses: $\Sigma m_\nu < 1.8 \text{ eV}$ [10].

Recent data from WMAP greatly improve this measurement. A key contribution is the fact that WMAP and 2dFGRS overlap in the wavenumbers probed. This allows a normalization of the 2dFGRS power spectrum from the WMAP data. The WMAP satellite also precisely determines Ω_m . Since depletion of power at small scales

is sensitive to the ratio of Ω_ν/Ω_m , a more accurate determination of Ω_m leads to a better bound on the neutrino mass. The ultimate result from combining data from 2dFGRS, ACBAR, CBI, and WMAP is $\Omega_\nu h^2 < 0.0076$ (95 % confidence level) [9]. The bound on $\Omega_\nu h^2$ places the bound masses $m_\nu < 0.23 \text{ eV}$ (3 degenerate Neutrinos, 95% confidence level). Note that using the WMAP data, [17] finds a more conservative bound of

$$m_\nu < 0.33 \text{ eV (3 Degenerate Neutrinos, 95% CL).} \quad (2)$$

The primary difference in the bounds is that [17] allows the bias factor to float in the analysis.

In the case where there are four neutrinos, the bound on neutrino masses is somewhat relaxed. As noted by [36], the bound on Σm_ν is anti-correlated with the value of the Hubble constant. On the other hand, limits on N_ν are correlated with Hubble constant. Playing these two effects against one another allows the weakening of bounds on m_ν for $N_\nu = 4$. For the 3+1 spectrum shown in Figure 2, again allowing the bias parameter to float, [17] finds a bound of

$$m_\nu < 1.4 \text{ eV (3+1 Neutrinos, 95% CL).} \quad (3)$$

This bound was derived assuming three degenerate active neutrino species. In the case where the neutrinos are not all degenerate, in principle one might expect the bound to be slightly modified, as the scale where free-streaming stops would be shifted. In practice, however, this has only a very small quantitative effect [37], so we neglect it in our discussion. Also, the above mass limit was placed assuming that the heavy neutrino has standard model couplings. These couplings determine when the neutrino decouples from thermal equilibrium. If the neutrino decoupled sufficiently early, it might have been substantially diluted relative to the active neutrinos. Consequently, it could contribute a relatively small amount to the critical density today. However, we do not expect this to be the case for an LSND neutrino. While one must be careful to take into account plasma effects, [38], that might keep sterile neutrinos out of equilibrium at high temperatures, these become negligible in time for LSND neutrinos to thermalize before decoupling. Reference [33] found that an additional sterile neutrino in a 3+1 scheme was nearly completely thermalized over the entire favored LSND mixing region. Since the neutrino ultimately decouples at temperatures of order 10 MeV, abundance of these neutrinos will not be diluted by the entropy produced at the QCD phase transition. This assures us that the limit of Eq. (3) is applicable for the heavy LSND neutrino as well.

Fitting the LSND result within a two neutrino oscillation picture requires (see Figure 1) a neutrino mass greater than the square-root of smallest allowed Δm^2 . This gives $m_\nu \gtrsim 0.45 \text{ eV}$. Comparing this with the bound on the neutrino mass in the 3+1 scheme, Eq. (3), one sees that the minimum LSND result is significantly squeezed by the large scale structure measurement alone. Taking

into account a full $3 + 1$ neutrino oscillation analysis, fully incorporating data from CDHSW and Bugey, we are forced into the small angle portion of the LSND allowed region. This means higher masses. At the 99% confidence level, the allowed region contains four islands, corresponding neutrinos with masses [16] (See Fig. 1)

$$m_\nu \gtrsim 0.9 \text{ eV}, 1.4 \text{ eV}, 2.2 \text{ eV}, 3.5 \text{ eV}. \quad (4)$$

All but the first of these conflict with Eq. (3), though the second is marginal. If, unlike the analysis of [17], one were to take a prior for the bias factor, the conflict would become stronger. So the LSND experiment is strongly constrained by large scale structure measurements alone.

If instead of the $3 + 1$ spectrum, we had chosen the inverted $1 + 3$ spectrum, the conflict would have been sharper. In the inverted case, the bound coming from large scale structure is stronger, (see Fig. 1), and the LSND islands are easily excluded.

It is interesting to note that the WMAP experiment also detected a relatively early re-ionization period, $z_{\text{reionize}} \sim 20$. This implies an early generation of stars responsible for the energy of re-ionization during this period. Early star formation disfavors warm dark matter, consistent with the above statements that neutrinos make up a small fraction of the critical density.

V. NEUTRINOLESS DOUBLE BETA DECAY

The limit on the neutrino mass from the combination of WMAP and 2dFGRS data is also interesting in the context of the neutrinoless double beta decay. The Heidelberg-Moscow experiment claimed a signal of neutrinoless double beta decay [13], which would indicate that neutrinos have Majorana masses. The relevant neutrino mass for the signal is the so-called effective neutrino mass $\langle m_\nu \rangle_{ee} = |\sum_i m_{\nu_i} U_{ei}^2|$. The nuclear matrix elements in [13] lead to the preferred range $\langle m_\nu \rangle_{ee} = (0.11\text{--}0.56) \text{ eV}$, while the reanalysis in [39] gives $0.4\text{--}1.3 \text{ eV}$ using a different set of nuclear matrix elements. This result does not require the presence of an additional (sterile) neutrino species, so the BBN limits need not apply. However, this high value of $\langle m_\nu \rangle_{ee}$ together with solar, reactor, and atmospheric neutrino data on mass splittings, require the three neutrinos to be nearly degenerate. In this case, the three degenerate neutrino bound of Eqn. 2 is appropriate, and the WMAP+2dFGRS data would therefore require $m_{\nu_i} < 0.33 \text{ eV}$, or $m_{\nu_i} < 0.23 \text{ eV}$, if the prior is taken on the bias factor. This large scale structure limit excludes the deduced range of the effective neutrino mass in [39] completely. However, using the largest values of the nuclear matrix element in [13], a window is still allowed. Also, a recent review of the evidence for neutrinoless double beta decay assigns a somewhat larger error for the matrix element, and the largest allowed

values of the matrix element could correspond to an effective neutrino mass as small as 0.05 eV [40]. So, the WMAP+2dFGRS constrains the claimed evidence for the neutrinoless double beta decay, but this statement is dependent on what is assumed about the nuclear matrix elements. Moreover, the WMAP+2dFGRS result has nothing to say about the Heidelberg-Moscow result if the neutrinoless double beta decay arises from a source other than Majorana neutrinos, such as supersymmetric models with R -parity violation [41].

VI. CONCLUSIONS

Recent precise cosmological measurements have given strong indications against the presence of an additional sterile neutrino in the range that would explain the LSND result. Bounds from BBN disfavor the presence of any additional neutrinos that do not decouple before the QCD phase transition. Large Scale Structure disfavors the presence of neutrinos with mass in the eV range.

It seems difficult to reconcile LSND with the cosmological data. We have already discussed the possibility of having a large pre-existing lepton asymmetry of $L \sim 10^{-2}$. Another possibility is to have CPT violation. In this case, the BBN constraint disappears, because no new light species are introduced. In addition, the large scale structure constraint is ameliorated, as only an anti-neutrino would need to be heavy, but not its CPT neutrino partner. However, KamLAND data, when taken in concert with data from Super-Kamiokande may disfavor this possibility [19]. The neutrino mixing result of LSND will be tested directly at the MiniBoone Experiment at Fermilab [42].

We also note that the cosmological data do not prefer the neutrinoless double beta decay in the mass range claimed by Heidelberg-Moscow experiment, unless the nuclear matrix element is very large.

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